EXPERIMENTAL VALIDATION OF THE SUPER ELLIPSE TYRE-GROUND CONTACT PATCH

VALIDAREA EXPERIMENTALĂ A MODELULUI SUPERELIPSEI PENTRU FORMA PETEI DE CONTACT DINTRE PNEU ŞI SOL

ROŞCA R.,CAZACU D., ŢENU I.,CÂRLESCU P.
University of Agricultural Sciences and Veterinary Medicine Iasi, Romania

Abstract. *The aim of this paper is to evaluate the validity of the Keller relationship for the case of 13.6 R38 tractor tire, using experimental data referring to the tire-ground contact patch. Using tire characteristics resulted in a shape exponent n=2.706 when the Keller relationship is used; the analysis of experimental data led to n=2,709±0.251. An analysis was developed in order to evaluate the goodness-of-fit between the contact patch area given by the super ellipse model and experimental data, which resulted in a Pearson correlation coefficient of 0.899 and in 75% of the model values being within the 95% confidence interval of the experimental data.*
Key words: contact patch, super ellipse, wheel load

Rezumat. *Scopul lucrării este de a verifica relaţia lui Keller pentru pneul de tip 13.6 R38, folosind date experimentale referitoare la pata de contact. Folosind relaţia lui Keller şi caracteristicile pneului rezultă exponentul n=2,706, în timp ce prin prelucrarea datelor experimentale am obţinut n=2,709±0,251.S-a realizat o analiză a corespondenţei dintre datele experimentale şi cele furnizate de către modelul superelipsei, rezultând un coeficient de corelaţie de 0,899, în timp ce 75% din valorile furnizate de către model se încadrează în intervalul de încredere de 95%.*
Cuvinte cheie: pată de contact, superelipsă, sarcină pe roată

INTRODUCTION

Many traction models assume that the tire - soil contact patch is symmetrical; Grecenko (1995) suggested that it has an elliptical shape and its area could be obtained by multiplying the product of the length and width of the contact area by a coefficient with values between 0.8 and 0.9. Hallonborg (1996) used a super ellipse model for the tire-ground contact area; the value of the positive exponent in the equation defined the shape of the contact patch.

McKyes (1985) developed a simple formula in order to compute the area of the contact patch, using the diameter and width of the tire.

Schjønning P., Lamande M. et al. (2008) also took into account the super ellipse to describe the symmetrical shape of the contact patch, with values of the exponent comprised between 2.45 and 4.51, depending upon tyre inflation pressure and model.

Saarilahti (2002) made a comparison of the footprint area given by different models and concluded that, for the same tire, the contact patch area given by different models had values comprised between 0.05 m^2 and 0.25 m^2.

Keller (2005) also considered the contact patch as a super ellipse and made measurements of the vertical stress below tires using compression cells. He developed an equation relating the shape exponent of the super ellipse to some tire characteristics and remarked a good agreement with the model taken into account (Diserens, 2002).

In this paper the experimental results obtained in a previous research activity were investigated in order to evaluate whether the super ellipse model for the contact area could be considered for the case of the romanian 13.6R38 tractor tires and, in the eventuallity of an affirmative answer, whether the Keller relationship, obtained for the same type of tires, is valid, taking into account that lower tire pressures and higher wheel loads were used in his experiments (70...150 kPa; 11...15 kN).

MATERIAL AND METHOD

The experimental researches were developed with the help of a specially designed test rig, as shown in fig.1.

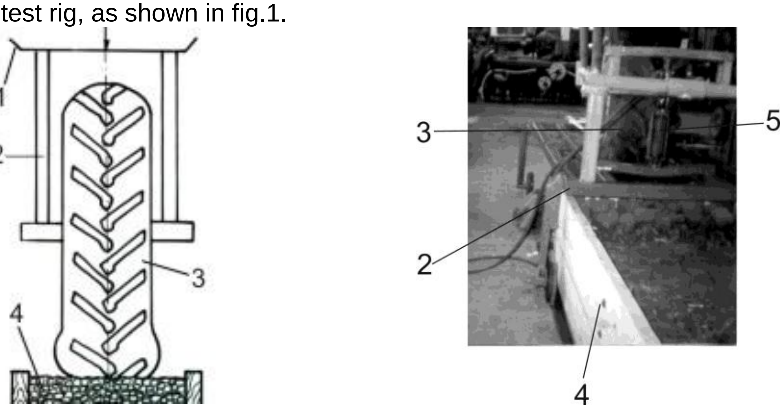


Fig. 1. The test rig
a) schematics; b) rear view; 1-loading tray; 2-frame; 3-wheel; 4-soil channel; 5-hydraulic cylinder

Wheel loading was achieved by adding weights in the tray (1); the contact area was recorded and measured for the testing conditions presented in Table 1.

The equation of the super ellipse is:

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1,$$

where a and b are the half axes of the super ellipse and n is a positive real number that determines the shape of the super ellipse.

In order to calculate the positive exponent in the equation of the super ellipse, the longitudinal and coresponding transversal dimensions of the contact patches were measured. A system of ecuations was then developed and then solved, thus allowing calculation of the major axis, the minor axis and the exponent involved in the super ellipse equation. The MathCAD software was used for solving the system of equations.

Table 1

Testing conditions and tire dimensions

Tire pressure [kPa]	Wheel load [kN]	Unloaded tire diameter [m]	Unloaded tire width [m]
200	5.39	1.58	0.367
	6.52		
180	5.39		
	6.52		

Three sets of measurements were made for each contact patch; the mean values and standard deviations were then calculated.

Keller (2005) found that the shape parameter of the super ellipse is dependent of the tire dimensions:

$$n = 2.1 \cdot (b \cdot d)^2 + 2,$$

where b is the tire width and d is the outer diameter.

The super ellipses were traced using a free super ellipse calculator and plotter, available at http://www.perbang.dk/superellipse/.In order to evaluate the goodness-of-fit between the shape of the real contact patch and the super ellipse, the corresponding areas were compared and the corresponding images were superimposed.

In order to evaluate the goodness-of-fit between model and experimental data in terms of contact patch area the following criteria were considered (Schunn&Wallach, 2005):

- percentage of points within 95% confidence interval of data (Pw95CI) – represents the percentage of model predictions that lie within the 95% confidence interval of each corresponding experimental data point;
- mean absolute deviation (MAD) – represents the mean of the absolute value of the deviation between each model prediction and its corresponding data point:

  $$MAD = \frac{\sum_{i=1}^{n} |m_i - d_i|}{n}$$

  where m_i is the model mean for point i, d_i is the data mean for each point i and n is the number of points being compared;
- mean scaled absolute deviation (MSAD):

  $$MSAD = \sum_{i=1}^{n} \frac{|m_i - d_i| \cdot \sqrt{m_i}}{n \cdot s_i}$$

  where m_i is the number of values contributing to each experimental data mean d_i (in our case m_i = 3) and s_i is the standard deviation for each data mean. A MSAD value of 1.5 means that, on average, the model is 1.5 standard errors off from the experimental data.
- Pearson correlation coefficient r^2.

RESULTS AND DISCUSSIONS

Analysis of the available contact patches according to the metodology presented in the previous section led to the results shown in table 2.

As expected, the effect of increased wheel load and lower tire pressure is the increase of the super ellipse major and minor axes and is also reflected in an increased area of the contact patch.

Table 2

Super ellipse characteristics (average values)

Item	Wheel load [kN]/Tire pressure [kPa]			
	5.39/200	6.52/200	5.39/180	6.52/180
major axis - 2a [m]	0.257	0.295	0.277	0.360
minor axis -2b [m]	0.200	0.210	0.207	0.208
n	3.076	2.449	2.432	2.391
contact area – A [m^2]	0.04918	0.05206	0.04461	0.05901

In the meantime the value of shape exponent was not significantly affected by the variations of wheel load and tire pressure; the mean value of the shape factor was:

$$n = 2.709 \pm 0.251.$$

Taking into account the outer diameter and width of the unloaded tire and using the Keller relationship led us to:

$$n = 2.706,$$

which means only 0.12% difference between the mean experimental value and the one given by the above mentioned formula.

The real contact patch areas are presented in table 3, while the results of the goodness-of-fit analysis are shown in figure 2 and table 4.

Table 3

Contact patch area (measured) [m^2]

Wheel load [kN]/Tire pressure [kPa]			
5.39/200	6.52/200	5.39/180	6.52/180
0.04908±0.00115	0.05393±0.00063	0.0406±0.00070	0.057±0.00104

From table 4 and figure 2 it is obvious that the measured data fit with the ones predicted by the super ellipse model, in terms of area of the contact patch: Pearson correlation coeficient is 0.899, the super ellipse model is 2.9 standard errors off from the experimental data and 3 of the 4 values predicted by the model are within the 95% confidence interval of the experimental data.

Table 4

Results of the goodness-of-fit analysis

Item	Value
Pw95CI	75%
MAD	0.0023
MSAD	2.901
r^2	0.899

The superimposed images of the super ellipses and contact patches are presented in figure 3 and also prove the goodness-of-fit between the calculated super ellipse contact patches and the real ones.

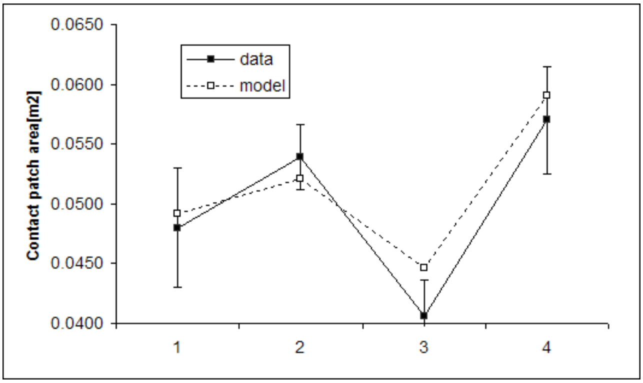
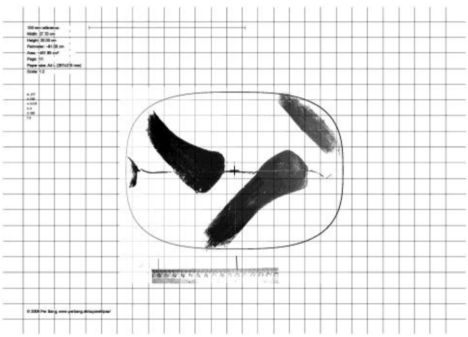
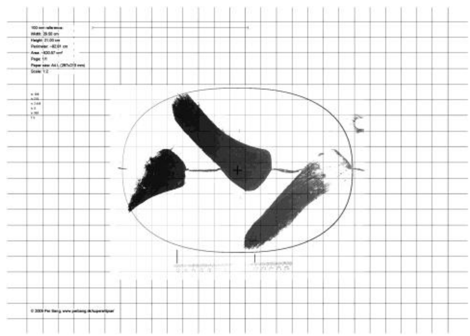


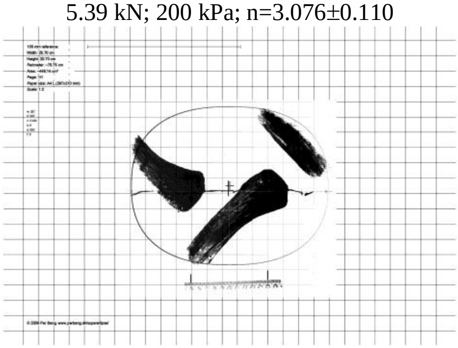
Fig. 2. Goodness-of-fit analysis



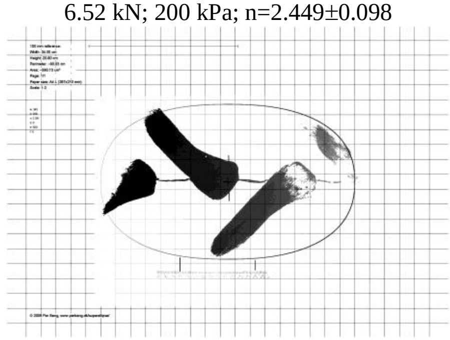
5.39 kN; 200 kPa; n=3.076±0.110



6.52 kN; 200 kPa; n=2.449±0.098



5.39 kN; 180kPa; n=2.432±0.121



6.52 kN; 180 kPa; n=2.391±0.117

Fig. 3. Superimposed images of contact patches and super ellipses

CONCLUSIONS

1. The super ellipse equation for the tire-ground contact patch was investigated in the terms of goodness-of-fit with real data;

2. The relationship developed by Keller was used in order to compute the shape factor involved in the super ellipse equation;

3. The difference between the calculated shape factor and the average one, obtained by interpretation of real tire-ground contact data, was les then 0.15%.

4. Evaluation of data referring to the area of the contact patch confirmed the validity of the Keller relationship with the Pearson correlation coefficient r^2=0.899 and 3 of the 4 values given by the super ellipse model being within the 95% confidence interval of real data.

REFERENCES

1. Bekker M. G, 1956 - *Theory of land locomotion*. The University of Michigan Press.

2. Bekker M. G., 1960 - *Introduction to terrain – vehicle systems*. The University of Michigan Press.

3. Diserens E., 2002 - *Determination of tyre contact area in the field using a mathematical model* (in german). FAT – Berichte nr. 582, Swiss Federal Research Station for Agriculture Economics and Engineering. Tanikok, Switzerland (cited by T. Keller).

4. Grecenko A., 1995 - *Tyre foot print area on hard ground computed from catalog value*. Journal of Terramechanics, 32, 325-333.

5. Hallonborg U., 1996 - *Super ellipse as a tyre-ground contact area*. Journal of Terramechanics, 33, 125-132.

6. Hambleton J., Drescher A., 2008 - *Development of improved test rolling methods for roadway embankment construction*. Research 2008-08, University of Minnesota, Minnesota Department of Transportation Research Services Section (available at http://www.lrrb.org/PDF/200808.pdf).

7. Keller T., 2005 - *A model for prediction of the contact area and the distribution of vertical stress below agricultural tyres from readily available tyre parameters*. Biosystems Engineering, 92(1), 85-96.

8. McKyes E., 1985 - *Soil cutting and tillage, vol. 7. Developments in agricultural science.* Elsevier, Amsterdam, p. 217.

9. Saarilahti M., 2002 - *Soil interaction model. Quality of life and management of living resources*. Contract QLK5-1999-00991 (available at http://ethesis.helsinki.fi/julkaisut/maa/mvaro/publications/31/soilinte.html).

10. Schjønning P., Lamande M. et al., 2008 - *Modelling effects of tyre inflation pressure on the stress distribution near the soil interface*. Biosystems Engineering, 99, 11-133.

11. Schunn C. D., Wallach D., 2005 - *Evaluating goodness-of-fit in comparison of models to data*. In W. Tack (Ed.), Psychologie der Kognition: Reden and Vorträge anlässlich der Emeritierung von Werner Tack (pp. 115-154). Saarbrueken, Germany: University of Saarland Press.